Data Structures

Data management systems are concerned with various aspects of data handling, such as:

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Storage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collection</td>
<td>Aggregation</td>
</tr>
<tr>
<td>Transcription</td>
<td>Update</td>
</tr>
<tr>
<td>Validation</td>
<td>Retrieval</td>
</tr>
<tr>
<td>Organization</td>
<td>Protection</td>
</tr>
</tbody>
</table>

Some useful goals for a data management system:

- Data must be represented and stored so that they can be accessed later
- Data must be organized so that they can be selectively and efficiently accessed
- Data must be processed and presented so that they support the user environment effectively
- Data must be protected and management so that they retain their value

Data structures help to achieve the first two goals.

A Data Structure is a way of organizing data so that it may be used efficiently. It is a class of data that can be characterized by its organization and the operations that are defined on it. With this definition, a data structure is no different from an object class or even a data type.
Records

The record is a simple yet fundamental type of data structure. A record generally contains an identifying field (key), e.g., EMPLOYEE, a data structure of type record:

<table>
<thead>
<tr>
<th>Telephone Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Job Title</td>
</tr>
<tr>
<td>ANALYST</td>
</tr>
</tbody>
</table>

What is a ... ?

Array within a record:
Record within record:
Record within record within array:

The value of the **key** uniquely identifies a record from within a collection of records.

A **record** is a finite, ordered collection of possibly heterogeneous elements, which may themselves be composite structures. The primary purpose of the record data type is to group together logically related fields. A collection of logically related record occurrences that are treated as a unit is called a **file**.
Lists

A list is a one-dimensional structure consisting of a collection of elements. If the elements are in contiguous locations, then the list is a linear list. [All vectors are linear lists.]
Arrays

An array is a finite set of ordered, homogeneous elements. A vector is a one-dimensional array.

Finite – a specific number of elements in the array
Ordered – elements are arranged so there is a first, second, etc.
Homogeneous – all elements must be of the same data type

Arrays are used when it is necessary to keep a large number of items in memory at the same time and to perform some operation(s) on them in a uniform manner.

Basic array operations:
- Declaration of the array
- Extraction of a given element value from the array (uses subscript)
- Storing a value in a given position in the array
- Searching for a target element
- Sorting the array

The word array usually (but not always) refers to a two- or more dimensional array. It is a vector whose elements are vectors. Multi-dimensional arrays are useful for organizing data which is dependent on two or more variables.

Examples:
Two-level array – Rating Table – Insurance rates by age and job classification:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18-34</td>
<td>23.50</td>
<td>25.25</td>
<td>27.05</td>
<td>52.90</td>
</tr>
<tr>
<td>35-39</td>
<td>24.00</td>
<td>35.75</td>
<td>27.55</td>
<td>53.40</td>
</tr>
<tr>
<td>40-44</td>
<td>24.60</td>
<td>36.35</td>
<td>28.15</td>
<td>54.00</td>
</tr>
<tr>
<td>45-49</td>
<td>25.30</td>
<td>37.05</td>
<td>28.85</td>
<td>54.70</td>
</tr>
<tr>
<td>50-54</td>
<td>26.30</td>
<td>38.05</td>
<td>29.85</td>
<td>55.70</td>
</tr>
<tr>
<td>55-59</td>
<td>28.00</td>
<td>39.75</td>
<td>31.55</td>
<td>57.40</td>
</tr>
</tbody>
</table>

Three-level array – Rating Table – Insurance rates by age, sex, and job classification:

<table>
<thead>
<tr>
<th>Age</th>
<th>Men [0]</th>
<th>Women [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Job Classification</td>
<td>Job Classification</td>
</tr>
<tr>
<td></td>
<td>Class 1 [0]</td>
<td>Class 2 [1]</td>
</tr>
<tr>
<td>18-34</td>
<td>23.50</td>
<td>25.25</td>
</tr>
<tr>
<td>35-39</td>
<td>24.00</td>
<td>35.75</td>
</tr>
<tr>
<td>40-44</td>
<td>24.60</td>
<td>36.35</td>
</tr>
<tr>
<td>45-49</td>
<td>25.30</td>
<td>37.05</td>
</tr>
<tr>
<td>50-54</td>
<td>26.30</td>
<td>38.05</td>
</tr>
<tr>
<td>55-59</td>
<td>28.00</td>
<td>39.75</td>
</tr>
</tbody>
</table>
Sparse arrays

e.g.:

```
  0 0 0 0 1 0 0 2 0 0
  0 1 0 0 0 0 0 0 0 0
  1 0 0 0 0 0 0 0 0 0
  0 0 0 0 0 0 0 0 0 0
  0 0 0 4 0 0 0 0 0 0
  0 0 0 0 0 0 0 2 0 0
  0 0 0 0 0 0 0 0 0 0
  2 1 0 0 0 0 0 0 0 0
```

The problem here is that there is a lot of wasted space. Consider, a 1000 by 1000 array, 1 million elements and, say, 1500 are nonzero. What is a more efficient way to represent this data?

How about an array of triplets, or 3 vectors:

<table>
<thead>
<tr>
<th>Row[i]</th>
<th>Column[i]</th>
<th>Value[i]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Assignment: Read in triplets (as records). Print out original matrix. Print our transpose. Do not at any time store the entire 8 x 10 array in main memory.
An array is an example of a static structure. *Static structures* allow for relatively sophisticated and often complex data organization. Their simplicity and easy access are offset by their “lackluster” existence, i.e., they lack any potential for change during their existence.

Static structures have the following characteristics:
- None of the descriptive information in the definition of the structure changes in any way during the lifetime of the structure.
- The data elements of an allocated structure are physically contiguous; a single block of storage is allocated to accommodate all the data of that structure.
- Interrelationships among data elements do not change during the lifetime of a structure.
Variable-length Arrays

The objective of variable-length arrays is to allow natural representation and manipulation of data used in certain applications, i.e., the data set must be allowed to grow and shrink in size (“dynamically”).

Accessing variable-length arrays by subscript is far less common and less important than with fixed-size arrays since each array element may change its position as the array changes in size. Instead, access is relative, e.g., “get the next element.”

Stack – a sequence for which elements may be inserted and deleted from one end only. We insert to and delete from the “Top.”
Queue – a sequence for which elements may be inserted only at the “Rear” and deleted only from the “Front.”
Deque (double-ended queue) – a sequence for which insertions and deletions may only take place at the ends.
Stacks

A stack is an ordered set of data items into which new items may be inserted and from which items may be deleted at one end only, called the Top of the stack. A stack is the most commonly used linear data structure.

Rules:
1. New items must be placed on top of the stack
2. Only items at the top of the stack may be removed
3. Stacks expand and shrink with the passage of time
4. The last element inserted must be the first element deleted (LIFO)
5. You only have access to knowledge about the items at the Top. You know nothing about any of the items below it unless you keep an auxiliary record.

When do we use a stack? When ordering doesn’t matter (e.g., memory locations).

Operations on a Stack:
- Push – adds an item onto the Top of the Stack, e.g., Push(s,x) adds x to stack s.
- Pop – removes the top element off the stack, e.g., x = Pop(s) removes the top element off stack s and moves its value to x.
- Test for Empty stack – since it is illegal to pop an item off of an empty stack (this is called ‘stack underflow’) we must first check for that condition, e.g., Empty(s) determines whether stack s is empty and returns the value true or false. Alt - IsEmpty(s).
- Peek – determines what the top item of the stack is without removing it. (Equivalent to a Pop and Push together.) e.g., Peek(s) returns the value of the top element of s without removing it from the stack.
- Top(s) is used to refer to or change the top value of the s.

Although an array cannot be a stack it can be a home for a stack. The stack will shrink and grow within the confines of the array.

Stacks are used when we want to process data in reverse order in which encountered – compilers use a lot. e.g., (1) matching parenthesis in arithmetic expressions. We can treat an arithmetic expression as a string, e.g., “( (A-B) /C-D* (E+F) )”

When stack is empty parens are matched. (Exercise: Given any string, write an algorithm which will check whether the parentheses match. Input string. Output Yes or No.)
(2) testing for proper nesting sequence

... if else if {
   while

(3) expression evaluation
For this expression (maybe in a string):   \( A + \frac{A+B}{D-C} - E \)

With these values:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

(4) Implementing recursive calls
Special case:
Suppose we want to implement two stacks, A and B. We may use two arrays of (say) size 100 each or a single array of size 200. Then the two stacks can grow and shrink dynamically within the confines of the array. Stack A grows ‘up’ from $s[0]$ while Stack B grows ‘down’ from $s[99]$.

<table>
<thead>
<tr>
<th>Stack A</th>
<th>... unused space ...</th>
<th>Stack B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s[0]$</td>
<td></td>
<td>$s[199]$</td>
</tr>
<tr>
<td>TopA</td>
<td></td>
<td>TopB</td>
</tr>
</tbody>
</table>

The advantage of this technique is that one stack may have more than 100 elements as long as the other stack has correspondingly few elements. Then, overflow occurs only when the sum of the number of elements in both stacks reaches 200.
Algorithms for Stack operations:

1. **PUSH**
   - **STACK[A]**
   - **TOP** = **MAX SIZE**?
   - **cout** << “No more room in Stack”
   - **Add 1 to Top**
   - **Stack[Top] = A**
   - **Return**
How to search a stack?

Suppose we wish to determine whether a particular element is in the stack.

We could Pop each element in turn, put it on another stack, and then restack. However, since we have implemented the stack as an array, it is simpler and more efficient to (temporarily) stop looking at it as a stack and look at it as an array just for the search.
Queues

A queue is an ordered set of items from which items may be deleted only at one end (Front) and into which items may be inserted only at the other end (Rear). This is a FIFO system. Examples: bank, barber shop, supermarket, bus stop.

Operations on a queue:
Create – declare and initialize
Insert – inserts an item at the Rear of the queue. Insert (Queue, X)
Remove – deletes an item from the Front of the queue. X=Remove(Queue)
Empty – Boolean function to determine whether the queue contains any values. IsEmpty(Queue)
Full – in array implementation, is the Queue full? We can’t insert into a full Queue.
Initially (in creation operation), \( \text{REAR} = 0 \) and \( \text{FRONT} = 1 \). Then, the queue is empty whenever \( \text{REAR} < \text{FRONT} \).

Problem: We reach the end of the "queue" (no more room in array) even though the beginning of the array is empty:

So, we cannot insert \( F \) even though the queue has room.
Solution #1:

Modify REMOVE so that when an item is deleted, the whole queue is shifted to the beginning of the array:

MOVE QUEUE (I) TO X,
PERFORM SHIFT VARYING I FROM 1 BY 1 UNTIL I IS GREATER THAN (REAR-1),
SUBTRACT 1 FROM REAR.

SHIFT,

MOVE QUEUE (I+1) TO QUEUE(I).

With this solution there is no need for FRONT since the first element of the array is always the beginning of the queue.
Empty queue is represented by REAR=0.

Disadvantages of Solution #1:

a) too inefficient - each deletion involves moving all the elements in the queue,

b) compromises the abstract meaning of the queue.
Better - we will use it.

**Solution #2:** view the queue as a circle rather than as a straight line, let the first element follow the last element:

**Note:** REAR is no longer determines empty queue.

Now set "FRONT" point to the element preceding the first element of the queue, then FRONT=REAR means empty queue.
Implementing Queue Operations

Creation:

CREATE-A-Q

DECLARATION OF VECTOR CALLED QUEUE IN PROGRAM CODE

FRONT = 100

REAR = 100

RETURN

Q-FULL = 'NO'

&

EMPTY = 'YES'

Test for Empty Q:

EMPTY-Q:

FRONT = REAR

IF

EMPTY = 'YES'

RETURN

FRONT = REAR

IF

EMPTY = 'NO'

RETURN

FULL-Q:

REMOVE:

EMPTY = 'YES'

IF

FRONT

IF

FRONT = 100

F

FRONT

IF

FRONT = 1

ADD 1 TO FRONT

X = QUEUE(FRONT)

Q-FULL

IF

Q-FULL = 'YES'

RETURN

FRONT = REAR

IF

EMPTY-Q

RETURN
When inserting a new element at REAR of QUEUE, must check for "overflow" (full array) after REAR value is modified.

We will return to STACKS, QUEUES, DE QUEUES in topic VIII.
Sorting and Searching
Linked Lists

Lists
Stacks
Queues
Multilinked lists
Nonlinear Data Structures

Graphs
Trees